# RECONSTRUCTING THE PHYSICAL CONTINUITY OF EVENTS

[09/2005 Note: This is a reconstituted 8/15/95 draft - This text was assembled from old disks for record purposes. I haven't read it through, but it represents the mid 90's work.]




## ABSTRACT

Measurements of change over time generally record physically continuing processes as a series of number pairs. Substituting mathematical functions for the data is one way to reconstruct an abstract continuity of the process to make a useful modeling tool. Another means of reconstructing the continuity of events is to apply an algorithm to the data that adds new points toward making a constructed continuity by iteration. This work uses the latter approach guided by physical principles derived from the conservation laws to study the underlying difference structure of recorded data sets. This appears to improve the resolution of the images of physical processes that can be obtained. Unexpected new information about the structure of natural systems is regularly found. The approach also provides an interesting new test for the validity of the conservation laws, i.e. whether naturally occurring processes display the specific time-series difference structure implied. The fundamental problems of the discontinuity of information and the difficulties of modeling the complexity of natural events are briefly discussed. The analytical software developed for the purpose is outlined and examples provided.


**Additional Key Words**: time-series, forecasting, systems, growth, modeling

# 1. GENERAL SYNOPSIS

The development of this method is intended to provide a new tool for investigating context and history dependent processes. It might apply equally to research on complex electrical, chemical, ecological, social or evolutionary processes and provides a way to further extend axiomatic scientific inquiry into the elaborate true stories told by individual events. It can not be used where no process of change is observable, such as with nuclear particle interactions, or where the available data is not sequentially ordered. What is essentially new in scientific content is the attention to individual events, and the construction of sequence continuities to without mathematical representation and replacement. To do this, with straight forward technique, data series are considered as a direct images of a natural event rather than as a statistical disturbance of a mathematical function. This requires a reconsideration of standard scientific methods of interpreting data established as far back as Lebesgue (Weiner 1949, p3) As it is for any other methodology the basis of validity for this work rests on its ultimate usefulness to others.

The key scientific problem addressed is the inadequacy of lists of data points to represent the complex continuities of the events we observe. The curves of data points are always made of dots and don’t tell us how the dots are connected. The traditional solution to this problem is to develop an equation that defines a curve passing through nearly the same dots, and then use that equation for predicting the course of similar events. This works well when reasonably simple equations provide reasonably reliable results. In many cases the circumstances are complex and every attempt at writing an equation produces very unsatisfying results. One of the reasons is that there are an unlimited variety of mathematical structures that can closely fit the data, and perhaps none which describe the natural process that produced the data. The problem is to find some way to directly look at the underlying process structures .

Here the object is to distill arithmetical difference patterns as they exist within a raw data set. These can be used as behavior models as is, or simplified in differential equations and used for modeling in the conventional way. The basic method is to insert new points in a time-series record that imply accelerations of change that are more likely than those implied by inserting new data points on the connecting straight line segments. Connected line segments describe alternating null and infinite accelerations of change, and the laws of energy conservation imply an absence of infinite accelerations. The type of continuity thus constructed can be called ‘organic’ because it is directly derived from the natural subject. As will be seen, it also develops a mathematical structure more complex than can be represented in mathematical functions.

The methods for inserting points are guided by the need to minimize the information content being added while developing a form of representation that is consistent with the conservation laws. An additional principle of reconstruction is added for data sets seen as multiple scales of description such as fractally organized and self-correcting (homeostatic) processes. These can be treated as the superimposition of underlying and superficial processes. Each many useful scales of description can be isolated for separate study by locating the inflection points of the implied data continuity.

One of the examples to be used here is a record of a simple physical motion, taking about a half second. The other, is the recorded history of U.S. economic product, covering about 130 years. Though this is an unfamiliar subject in the literature of the physical sciences, it provides an excellent example of a complex unrepeatable individual event for which new methods of investigation could be very useful. There is plentiful data and various well developed alternative approaches for imaging the structure for comparison.

That the subject itself is a familiar one is also important. Direct experience of any subject is necessary for contextual understanding. In many cases this approach relies more heavily on contextual understanding than others. The object is to identify natural structures in dynamic and locally unique events where structure is normally very difficult to find. This quickly and directly takes you to questions that are just a little beyond the limits of our understanding. In the case of the economic record the principle continuity derived identifies an unexpected but understandable large scale pattern underlying the historic direction of the growth rate. That this result is not a coincidence of this application is supported by using this method to duplicate a controversial discovery made recently with an advanced econometric method, (Young 1994)[i] . This supplemental example is presented in Appendix B along with some a discussion of other recent topics in the statistical analysis of time-series.

Appendix A describes the software developed and used for this application. Correspondence and good quality data for experimental use describing any complex event would be welcome.

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[i]Concerning a close dynamic relationship between the reconstructed first and second derivatives of long-term growth and unemployment trends using a non-linear variable parameter spectral analysis method/

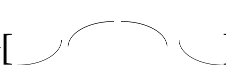
# 2. INTRODUCTION

All scientific work rests on observation. In conventional scientific research observations are collected and examined and then a hypothesis is formed and tested against the observations, usually in terms of how closely a mathematical function statistically fits the data. The subject here concerns the step in this process prior to constructing or testing a mathematical model, the direct examination of the data. Here the data is considered as a direct image of the subject, not a statistical disturbance of some underlying function, and the object is to extract improved images of the natural process structures from it.

The origin of this work was an effort to investigate a family of complex events that were individually coherent but unpredictable, a study of air current networks in natural micro-climates(Henshaw 1978). When local room to room scale, or larger air currents develop, the pathways are established as smaller air currents of various scales wander onto the same paths and then coalesce to form a climax current structure. That climax structure may remain stable until the remote thermal imbalance that drives it is depleted. Then it breaks up.

In the field of fluid dynamics engineers now have specialized computational tools[i] that could be very helpful in micro-climate study and there is some interesting new academic work on related micro-current process structures.[ii] Still, there is little to be found addressing the common types of natural air currents or their evolutionary processes for which simpler methods were then being successfully used for both experiment and design. The work presented here comes from that experience as an effort to describe how relatively simple techniques can yield some degree of structural knowledge of complex natural systems currently beyond the reach of the conventional scientific methods.

The point at which that study became productive was when the classic sequence of exponential and asymptotic progressions from origin to decay, the permutations of exponent and parameter sign reversals:

[++]/[-+]/[+-]/[--], or graphically [  ] (1.1)

[i]CFD, Computational Fluid Dynamics has been applied to natural convection problems with some success in various fields including building climate modeling and design.
[ii] Applied Scientific Research V51 1993 "Advances in Turbulence"

were recognized as corresponding to distinct periods of organizational change . These patterns appear in the growth and decay of air flows in evolving current structures and also serve as useful markers in the progressions of individual emerging systems of other kinds. In air current structures the organizational changes during these periods could be characterized as using the names: C*onnection, Unification, Separation and Passing,* comprising a complete evolutionary history from beginning to end. These descriptive terms for the phases of organizational change were chosen partly to form the acronym, *CUSP,* because each growth period is a process of leaving one organizational state and approaching another. There are many cautions needed when using these terms and markers in general application and a lengthy discussion is not included here. Among the principal difficulties confronted is the individual uniqueness and transience of any occurrence, and that our information about them is usually sketchy.

This work fits most directly into the main stream of scientific research as a variation on current methods used in the areas of computerized pattern recognition, forecasting and simulation, and as a new tool to add to the study of time-series. The first step in reconstructing a natural process for computer simulation is to "derive a difference equation from finite difference approximations of the derivatives"(Smith, 1987 p143) What this work provides is a disciplined method based on physical principles for enhancing the finite difference approximations of the time-series data. The results may be used to assist in developing a better modeling equations or in place of them or to identify structural relationships directly.

The general scientific texts on time-series studied for this paper [i] begin the study of a data set with statistically mixing adjacent data points, usually by some kind of averaging. It is generally considered that scientific study must treat individual records of events as statistical representations of a larger class of events and their universal mathematical function[ii]. Not only data from different series are treated statistically, but also the data points representing different times in the same series. Beyond consideration for the uncertainty of the measurements, this latter practice does not seem to have a basis in

[i](Weiner 1949)(Otnes & Enochson 1972)(Meisel, 1972)(Chatfield 1975)(Cuthbert & Wood 1980)(Harvey, 1981)(Kmenta 1986)(Smith 1987)(Evans 1988)(Casti 1989)(Mills; 1990)(Bendat, 1990)

[ii](Weiner 1949) p1 'Time series are sequences...studied with respect to the statistics of their distribution in time" p3 "The events which actually happen in a single instance are always referred to as a collection of events which might have happened" p11 "Without at least an approximate repeatability of experiments, no comparisons of results at different times are possible, and there can be no science. *That is, the operators which come into consideration are invariant under a shift in the origin of time.*"(original emphasis)

theory, though it may be expedient sometimes. Especially when considering the unrepeatable aspects of individual events, it seems more appropriate to consider the data as a direct image of the subject process, i.e., of what actually occurred, rather than as an image of what might have occurred. When considering a unique individual record, and time as a process rather than as a location, the averaging of adjacent points blurs the image. Instead of first 'softening' the variations from point to point, here the equivalent first step would be to sharpen the image, based on assumptions such as physical continuity, by inserting new points where one could realistically expect them.

Unlike the traditional approach to time-series where fitting a function to the data is the primary object, here the approximation and replacement of a data set with a formula is consciously avoided. The principle purpose is to preserve the details of events so that a clear image can be constructed. Here, instead of an equation, the raw data itself is the preferred mathematical analogy of the subject process and the object is to extract information from it. The results of analysis are thus homoeomorphic representations of the behavior, rather than paramorphic representations[i] as are the normal practice in the physical sciences. The resulting objects, of course, also need to be handled differently.

It was fundamental to the construction of this method to discount the normal supposition that natural processes are mathematical in nature, a presumption that seems to date back to Aristotle[ii] This grew out of an interest in what could be learned about periods of transition between mathematically regular states, when any preceding or following mathematical structures would be undefined. Restricting a study to mathematical descriptions inevitably excludes consideration of these transitional processes and periods.

The use of mathematics for describing natural events is problematic in other ways as well. Mathematical functions are generally either defined throughout all time or begin and end abruptly or with uncertainty. Certainly our observations begin and end abruptly or with uncertainty, but that is not inconsistent with natural processes being strictly temporal and developmental. It could be just that there are always beginnings and endings beyond the sensitivity of our measures. The structure of mathematical functions also generally remains constant throughout their range whereas the processes of a individual events can be in constant flux. Mathematics is also difficult to make

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[i](Harré 1972, p 182)

[ii]The practice of representing nature as mathematics appears to have originated with Aristotle, who wrote that "it is manifest then that time is a number....and is continuous, for it is of what is continuous" and "....It is because the magnitude is continuous that the change is too" Aristotle's Physics, Book IV 220a24 & 219a10

deeply nested (hierarchical) in its structure as the systems of nature tend to be. This structural dissimilarity between mathematics and natural behaviors can conceal many phenomena, especially when mathematics is used to exclusion for describing events. For example, the use of mathematics almost inevitably constitutes natural processes as exogenous (i.e. 'following' a formula), when, in many cases, and perhaps in general, the structures of nature might more reasonably be considered as endogenous (i.e. self-evolving).

The difficulty of representing transitions between separate states of behavior with mathematics can be intuitively understood from the common experience of driving on a road with ungraduated curves, resulting from the surveyor laying out the road by splicing one geometric curve to another without any transition. Staying in the driving lane requires sudden jerks of the wheel. In mathematical description, these abrupt changes of mathematical definition are identified as points with infinite rates of change in the derivatives. A kind of seamless *organic* continuity that is not easily described, rather than abrupt discontinuities between states of simple description, is much more characteristic of natural processes[i]..

Among the subjects that will not be addressed at length are the specific mechanisms of natural organizational growth and decay. It appears that these frequently involve cascades of events, interacting with contextual located remnants of related past events and other varieties of distributed and circumstantial causation. The approaches to this subject of general systems theory[ii]and the physical science of dynamics concerning complexity and chaos[iii] take different approaches to the subject but there is much in common in terms of the particular problems of nature recognized. It is hoped that the techniques of reconstructing continuities presented here will be significantly useful for further identifying the problem for both of these disciplines, as well as making a contribution to the art and science of time-series modeling.

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[i]The positivist route to validating generalizations can not be met for continuity. Continuity is a presence of information about connecting processes, and discontinuity an absence of information. Since all measures and observables begin and end with an absence of information neither continuity nor discontinuity of natural processes are *refutable*, thus the weaker test of whether a principle of continuity is *useful* is adopted here.
[ii]see (Van Gicch 1978)(Miller 1978)(Meadows, Meadows & Randers 1992)
[iii]see (Looss & Joseph 1980)(Gleick 1987)(Nicolis & Prigogine 1989)
(Lewin 1992)(Kauffman 1993)

# *3*. A PRINCIPLE OF PHYSICAL CONTINUITY

The principle of energy conservation can be used to suggest a new general principle of continuity in physical motion, and by implication, for the general continuity of change in physical systems. This will then provide both a basis and a test for the presumption of continuity for investigating the records of individual events.

A simple example of where the test would apply is in the motion of colliding billiard balls. There is an easily described pattern of motion before and after the collision that can be calculated from a list of the times and locations of the balls. Without an exceptionally high speed recorder, however, a discontinuity of description would appear at the transition from one steady state to the other, the gap in the record during which the balls are actively colliding. If physical continuity is assumed the data is considered as describing a process that does not have infinite accelerations and can be explored for concrete evidence of the transitional process that presumably must have occurred. The general demonstration and usefulness of the continuity principle follows from identifying a particular form of exponential-like acceleration that theoretically must and apparently does occur in natural processes of most kinds.

The theoretical demonstration begins by presenting the conservation of energy as related to the conservation of momentum and reaction forces in an infinite hierarchy of physical laws where each law can be stated as the derivative and integral of others. The three basic laws[i] are stated in the first three lines of equations in table 3.2 below. In reverse order, the first three statements under the heading of "Unified Form" are created from their "Conventional Form" by substituting derivatives of distance (**s**) for acceleration (**a**) and velocity (**v**), and in the case of energy conservation, the conventional term $\frac{1}{2}v^2$ is replaced by the integral of its derivative ($\int v \cdot dv$), a quantity having the same derivative rank as distance (**s**).

The general principle of continuity is then derived by successively differentiating as a limit and concluding that the sums of all derivative rates of mass motion within a closed system are conserved. This yields a curious but firm statement parallel to the conservation of energy, momentum and forces. Without presupposing any particular

[i]For example see (Miller 1967) pp 114-150

interpretation one can conclude that the mathematical laws of conservation imply that the physical beginnings and endings of motion are conserved as well.

To this point little definition has been given to the behavior of individual masses, only to the sum of mass accelerations within a closed system. Now we will consider how individual masses are further constrained in the manner in which they can accelerate and then derive an observable form of acceleration which, by implication, individual events must incorporate.

A corollary of the conservation principle, shown under the heading “Limits of Motion” in table 3.2 establishes that all mass accelerations must be finite, starting with the basis from relativity that velocity can not exceed the speed of light $\mathbf{v} < \mathbf{c_1}$ (3.2.2c). If there were a mathematical function fully describing a change in movement every derivative rate of the function at every point in time (each $\boldsymbol{r_n}$) must also be less than some constant and therefore finite , as follows:

For **i**, **j**, **l**, **n** - integers; **c**, **k**, **p**, **q**-constants; **W** - work; **F** - forc

If, at the n’th derivative level $\boldsymbol{r_n} < \boldsymbol{c_n}$ 3.01

in any finite period $\boldsymbol{r_n} = \boldsymbol{r_{n+1}} \cdot \Delta \boldsymbol{t} + \boldsymbol{k_n}$

by substitution $\boldsymbol{r_{n+1}} \cdot \Delta \boldsymbol{t} + \boldsymbol{k_n} < \boldsymbol{c_n}$

and $\boldsymbol{r_{n+1}} < \left(\boldsymbol{c_n} - \boldsymbol{k_n}\right) \div \Delta \boldsymbol{t}$

let $\left(\boldsymbol{c_n} - \boldsymbol{k_n}\right) \div \Delta \boldsymbol{t} = \boldsymbol{c_{n+1}}$

so that

at the n+1 derivative level $\boldsymbol{r_{n+1}} < \boldsymbol{c_{n+1}}$ 3.02

### *3.1 Basic Formulas of Work*

| | |
|---|---|
| $\boldsymbol{W} = \frac{1}{2}\boldsymbol{mv}^2 = \frac{1}{2}\boldsymbol{m} \cdot \left(\frac{\boldsymbol{ds}}{\boldsymbol{dt}}\right)^2$ | Work, Energy of accelerating a mass to a velocity |
| $F = ma = m \cdot \frac{dv}{dt} = m\frac{d^2s}{dt^2}$ | Force corresponding to acceleration for a mass (a first derivative of Work) |

### *3.2 Laws of Conservation and Continuity*

| | Conventional Form | Unified Form | Limits of Motion |
|---|---|---|---|
| 1. Conservation of Energy<br>• sum of energies is constant<br>• 0 derivative level | **a.** $\sum_{\boldsymbol{i}} \frac{1}{2} \boldsymbol{m_j} \cdot \boldsymbol{v}^2{}_{\boldsymbol{j}} = \boldsymbol{k}$ | **b.** $\sum_{\boldsymbol{i}} \boldsymbol{m_j} \int \boldsymbol{v_j} \cdot \boldsymbol{dv} = \boldsymbol{k}$ | **c.** $s_j < c \cdot t + k_1$ |
| 2. Conservation of Momentum<br>• sum of momentums is zero<br>• 1st derivative level | **a.** $\sum_i m_j \cdot v_j = 0$ | **b.** $\sum_i m_j \frac{ds_j}{dt} = 0$ | **c.** $\boldsymbol{v_j} < \boldsymbol{c}$ |
| 3. Equality of Reaction Forces<br>• sum of forces is zero<br>• 2nd derivative level | **a.** $\sum_i m_j \cdot a_j = 0$ | **b.** $\sum_i m_j \frac{d^2 s_j}{dt^2} = 0$ | **c.** $a_j < c_2$ |
| 4. Un-named<br>• sum of 2nd accelerations zero<br>• 3rd derivative level | | **b.** $\sum_i m_j \frac{d^3 s_j}{dt^3} = 0$ | **c.** $r_j < c_2$ |
| 5. Principle of Continuity<br>• sum of mass accelerations zero<br>• n'th derivative level | | **b.** $\sum_i m_j \frac{d^{n2} s_j}{dt^n} = 0$ | **c.** $\boldsymbol{r}_{\boldsymbol{j_n}} < \mathbf{c}_{\boldsymbol{n}}$ |

### *3.3 Individual Events*

We now consider some individual mass (**m**)within the system **S**. If **m** is at rest, for it to develop a positive velocity its acceleration must have been positive for a finite period. The same then is true for it to achieve a positive acceleration, and for every other underlying acceleration. Thus for there to be a change from rest to motion, there must be a finite period during which all underlying rates are positive. One class of mathematical functions of this kind are the exponentials. The polynomial form of an exponential directly results from the successive integration of a constant.

Assuming for some large **n** the n'th derivative rate of change is greater than and less than some pair of constants:

$$\boldsymbol{l_n} > \boldsymbol{r_n} > \boldsymbol{k_n} \qquad 3.30$$

the integral of the lower bound constant, including a lower bound integration constant gives:

$$\boldsymbol{r_{n-1}} = \int \boldsymbol{r_n} = \boldsymbol{k_n} \cdot \boldsymbol{t} + \boldsymbol{k_{n-1}} \qquad 3.31$$

in general, for $\boldsymbol{i}$ less than **n** the polynomial expansion of the integrals of $\boldsymbol{r_n} > \boldsymbol{k_n}$

$$\boldsymbol{r_{n+i}} > \frac{\boldsymbol{k_n}}{(\boldsymbol{n}-1)!} \cdot \boldsymbol{t}^{\boldsymbol{n}-1} + \frac{\boldsymbol{k_{n-1}}}{(\boldsymbol{n}-2)!} \cdot \boldsymbol{t}^{\boldsymbol{n}-2} + \ldots \boldsymbol{k_{n-i}} \quad 3.32$$

As $\boldsymbol{n}$ **and** $\boldsymbol{i}$ **n** and **i** are increased as a limit this will match the general polynomial form of the exponential function when all values of the series of constants $\boldsymbol{k_n}$ are real and positive. Since during some finite period at the transition between rest and motion $r_n$ must be positive and finite for arbitrarily high all $r_n$ must be positive and finite, there will be constants $k_n$ and $l_n$ values of **n**, there will be a series of constants $p_n$ and $q_n$ close to , above and below, for which the limit of 3.32 describes exponential functions closely bounding the physicalleast upper bound and greatest lower bound exponential functions bounding the curve during the period. An ideal derivative reconstruction algorythm would define the least upper an greatest lower bounds for the derivatives implied by a given set of data.

The implication is not that energy conservation requires a 'regular' exponential in every beginning and ending of change. No condition has been placed to require all integration constants to be equal, for example, or even for the parameter values to be actually constant. One additional interesting property of exponentials seems to be preserved none-the-less, and may be demonstrated elsewhere. That is that the ratio of the successive derivatives approaches a constant when the independent variable (expressed above as '**t**' for time) has a unit value. This indicates a way in which the separation trajectory during the periodof the upper and lower bound curves can be interpreted as being arbitrarily small. Thus it would appear that a change from any state of rest requires a complex hierarchy of accelerations that fit at least approximately the form of an exponential curve.

Exponential, or exponential-likThat accelerating acceleration and decelerating deceleration are required for continuity in physical events should be no particular surprise. Such exponential, and complex exponential-type progressions are readily observable in the beginnings and endings of events of all sorts. That this is difficult to describe theoretically and seemingly quite complex suggests a need for further work.

What is mathematically curious is that this describes what seems to be a mathematical impossibility unless physical processes are unbounded in time. EAnother problem concerns the fact that exponentials and other functions can be made to very closely fit the observable regular transitional processes, but can not do so unless they aremeasures of change, but do so with values defined throughout all time or have a discontinuity at their beginning and ending pointswith discontinuities at their points of beginning and ending. It is a simple proof that any continuous single valued function that contains a constant (zero slope) segment in any of its derivatives, or that is made of a spliced sequence of continuous functions must contain infinite derivatives. All these forms of equations are thus ruled out as candidates for describing events where the conservation laws apply. What you might call a naturally continuous function appears to be quite hard to define. Such functions may be ultimately un-writable in fact, but do , however, appear to be numerically constructable with writable rules.This author's expectation is that the implied continuous paths of physical measures will remain difficult to define mathematically, and greater reliance will be made on iterative constructions of continuity based on definable rules.

As mentioned in the introduction, because of the gaps in information at the limits of observation it probably can not be directly observed whether transitions from and to steady states are or are not continuous. The demonstration is then left to the weight of the evidence and the usefulness of either presumption. Ordinary mathematical functions can not describe continuous transitions between different formulations, and the question would be whether and how natural processes do, and what kind of mathematics to develop to help us understand what happens. If it were widely recognized that measures of transitional events normally include distinct starting and ending periods when the observable underlying rates of change are all of the same sign, then the principle of continuity and the laws of conservation would be re-confirmed.

# 4. ANALYTICAL METHODS

The method of data analysis used here treats time-series as direct but incomplete images of a physical processes. The rules applied to a data sequence serve to partially complete the image and expose layers of implied underlying structures in the accelerations of physical processes. Unlike a conventional search for mathematical structures in nature, no mathematical statement of the natural process is required, only confirming identification of the newly exposed structures and events by other means.

These methods are not formalized and work is needed to develop fully reliable analytical rules, terminology and measures of confidence. An algebra of natural sequence analysis seems to be needed that might be similar to the algebra of difference equations developed to approximate solutions to systems of differential equations. The algebra of difference equations translates mathematical functions from the form of continuous but unsolvable mathematical relations into the form of discrete solution sets. An algebra of natural sequence analysis would start with discrete solution sets and establish methods for extracting the implied derivative[i] structures.

Another practical difference from the normal scientific method is the manner of using statistics. In conventional work you propose a mathematical equation and then test its statistical fit with the data as a measure of its validity. If adjusting the constants of the function allows it to fit the data well, or at least better, the mathematical relation may be substituted for the behavior in future discussions. The normal measures of fitting a data set are based on a least squares regression where the measured values of the data at different times are considered to be stochastic (random) distortions of the function being evaluated. The probability that a reconstructed continuity has accurately identified the true inflection points of the underlying process can not be measured that way, thought it is still a valid issue worthy of study.

In the analysis of an implied continuity it is generally assumed that the mathematical function which is represented is at least too complex to be written. The validity of any internal structure of the data exposed is primarily demonstrated by the repeated appearance of discovered structures from multiple views and independent confirmations by other means. The approach is therefore more archeological, rather than military, in its conquests. Every scrap of data is important, and the analytical tasks are a kind of filtering and distilling, more than the more familiar approximation and replacement which characterize conventional science.

Because so much trust is put in the data, it is especially important whether or not it is reasonable to infer the continuity of a subject for which you have only an intermittent record. Without that assumption a data sequence can not be taken to describe an implicit continuous phenomenon, nor would there seem to be any other necessary relationship between its separate points. When continuity can be assumed, one can then proceed to identifying and separating different scales of continuity in the data to more clearly

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[i] Note that this is a mixed use of the term 'derivative' as defined in the calculus. The derivative structure in this case is implied by patterns that appear in successive differences in a finite real valued series. One might prefer to reserve the use of the term derivative exclusively for continuous mathematical functions rather than allow it to apply to physical systems and finite series as well. However, the author feels uncomfortable with discussing the 'differences' of a series or having to always use terms like 'the underlying acceleration patterns' of natural systems, and so 'derivative' will be used somewhat broadly here to refer to implied or potentially constructable underlying continuities of acceleration. As usual the terms 'differencing' and '1st and 2nd differences' will be used when referring to specific arithmetical operations. The term 'integration', however, will be used interchangeably due to the difficulty of using 'additions' or

display the natural process structures that may be present. The kind of information being sought by this method is often so very hard to gather by fitting conventional equations, if continuity analysis provides only small amounts of additional information the results could be quite valuable.

In this study a general graphical database software, AutoCAD[i], was used in conjunction with AutoLisp[i] programming to handle data sets as graphical objects to which analytical calculation functions can be applied. The graphic figures that follow are direct plots of the data sets used and the results of applying the various analytical operators. More information on the program operators used and programming details is provided in Appendix A. The basic procedure is to import a time series data sequence as a graphical database object called a polyline, and then to use analytical functions to create other time-series curves from it. These functions generally proceed from one end to another evaluating the relationships between a number of adjacent points. As each new curve is created it is tagged with the names of the operations applied to make it, so that a record of its history of development is maintained and the steps reproducible.

The limited technical objective of the current paper is to present a method of reconstructing the continuities of time series data to make them sufficiently differencable to expose unexpected new information about the subjects being considered. This seems to have been achieved to a satisfactory degree derivative reconstruction (dr) using interpolation and smoothing based on sequentially minimizing the fourth derivative to reconstruct the implied continuities, and inflection point bridging to separate different scales of data fluctuations.

A preliminary statement of the analytical principles used is as follows:

**(4.1)** The addition of intermediate points in a data sequence that does not introduce unnecessary fluctuations and minimizes the implied range of accelerations probably provides a more accurate representation of the original continuity than the original data (the basis for derivative interpolation).

**(4.2)** When the fluctuations in a series are of the scale of the

'periodic summation' of a series, or other like terms that would be more specific to a use with finite series.

[i]AutoCAD and AutoLISP are registered trademarks of AutoDESK inc.

statistical accuracy of the measure itself then adjusting points to minimize the implied range of accelerations between adjacent points probably provides a more accurate representation of the original continuity than the original data (basis for derivative smoothing).

**(4.3)** A subset of a sequence that eliminates fluctuations describes a larger behavioral scale. Fluctuations about a trend tend to cross the trend at their points of maximum slope. Connecting these points, will produce a larger scale of description with the minimal accelerations necessary. (The basis for inflection point bridging).

Following these principles operators were made for successively constructing new points for the sequence to locally equalize the 3$^{rd}$ derivatives on either side, and for trend line bridging to construct larger scale descriptions by connecting points with implied zero 2$^{nd}$ derivatives.

Third derivative interpolation and smoothing were found to be quite successful in reconstructing a continuity. What they produce are the points a curve would go through to arrive at the same end with minimal underlying accelerations. This accomplishes remarkably strong local smoothing with much less distortion multi-point averaging,. Third derivative smoothing by the algorithm used (see Appendix A) primarily regularizes the accelerations in the path of the curve, not its destinations. With some care, derivative interpolation and smoothing make even short data sequences repeatedly differencable, with a good likelihood of meaningful results.

Trend bridging was developed initially to thread through fluctuations in a way that mimics the trends that can be seen by intuitive visual inspection of an otherwise irregular curve. The function developed was soon found to be especially useful for separating the fluctuations and trends of homoeomorphic (self-correcting) processes. For these processes there is sound physical reason for the rates of excursion from the norm to reverse, like the reaction forces on a vibrating string, as the position of the subject crosses the point of equilibrium. Where other physical assumptions are more applicable other analytical methods would need to be used.

Derivative reconstruction serves somewhat like a micro-scope for natural process structures for its ability to expose otherwise hidden smaller scale events. Inflection point bridging might be referred to as a macro-scope for its ability to expose otherwise hidden larger scale structures. Like other magnification tools, issues of focusing and limits of resolution are significant concerns.

In addition to these essential operators a variety of support functions were developed including, basic differencing and integration functions, a general graphic calculator accepting curves as variables, and utilities to increase and decrease the number and spacing of points. A directional bias is sometimes introduced by including new points with the adjusted values of old points in subsequent evaluation sets. That bias was minimized by combining both forward and backward scans of the data. End points are generally kept in the resultant curves, even when the rule primarily concerns points in the middle of a range. This was done by adjusting the algorithm near the ends of the sequence to either operate with fewer points or to use projected points following a damped derivative projection from adjacent points in the series.

End points are kept in the sequence for two simple reasons. Dropping points whenever there are too few preceding or following points to perform a particular calculation causes a rapid decline in the number of points available to consider. It may be less than ideal, but it is also more accurate to include end points treated by different rules, presenting degraded results toward the ends of the curves, than to portray the event under consideration as ceasing during those periods. It should be noted that because end points were treated differently they therefore also need to be interpreted differently. In presenting graphs of the results the regions of less confidence near the ends may be drawn more lightly to suggest that the quality of the information fades at the ends of the curves.

## 5. IMPLICATIONS OF PHYSICAL CONTINUITY

The conventional scientific meaning of the term ‘continuity’ concerns the proximity of mathematical points in relation to each other within the definitions of the calculus Here the concern is not with mathematical continuity, but with the physical continuity of natural processes and their investigation using the implied continuity underlying time series data. To the knowledge of the author, physical continuity has not been specifically treated as a scientific subject before, only as a concern of metaphysics.

The study of physical continuity retains the familiar intuitive meanings of ‘connected’ and ‘inseparable’, but because of its application to implied structures, which can not be fully defined, some of the completeness of the definition is lost. The appropriate resort seems to be to define the term by reference, as a property of that still poorly understood way nature has of connecting things. Wherever we look for smaller scale connecting events in-between the ones we already know about, to the limits of our observation, we tend to find them. .

One of the simplest and best uses of this method is for identifying implied discontinuities in natural processes, as shown in figure 1. Here a constructed curve gives the appearance of representing a smooth process of transition from beginning to end. The actual curve shown was formed by mirroring a constructed exponential curve segment about the point where it reaches a unit slope. Taking the first and second differences of the composite curve clearly displays the splicing points as discontinuities of description, where the formula abruptly changes, and identifies the original appearance of continuity as false.

Such a curve may well offer a close scalar approximation to the measures of its subject process and, by conventional rules, provide an appropriate representation of it . Because of the abrupt transition from one mathematical description to another however, structural irregularities are introduced that are inconsistent with physical continuity. One might also observe that these discontinuities re are located at points in the curve of particular interest, the points where the subject process would seem to be changing from one relatively constant state of behavior and description to another. This is often where normal techniques break down, arbitrary patches need to be made, and the processes of transition are concealed within a gap in description.

A difference algebra of sequences that would avoid some of these problems would parallel the established algebra of the calculus. The term ‘derivative’ is conventionally considered in terms of continuous mathematical functions according to the definitions of the calculus[i]. Here the simple derivative of a sequence is a sequence of the successive differences.

**(5.1)**

$$\Delta \boldsymbol{S} = \left( \boldsymbol{s}_1 - \boldsymbol{s}_0 , \boldsymbol{s}_2 - \boldsymbol{s}_1 , \boldsymbol{s}_3 - \boldsymbol{s}_2 \ldots\ldots \boldsymbol{s}_{\boldsymbol{n}} - \boldsymbol{s}_{\boldsymbol{n}-1} \right)$$

where S is a sequence of **n** values

$$\boldsymbol{S} = \left( \boldsymbol{s}_0 , \boldsymbol{s}_1 , \boldsymbol{s}_2 \ldots\ldots \boldsymbol{s}_{\boldsymbol{n}} \right)$$

The simple derivative of a sequence with respect to another sequence(perhaps time or any other reference) is a sequence of ratios of the successive differences of the two.

**(5.2)**

$$\frac{d\boldsymbol{S}}{d\boldsymbol{T}} = \left( \frac{\boldsymbol{s}_1 - \boldsymbol{s}_0}{\boldsymbol{t}_1 - \boldsymbol{t}_0} , \frac{\boldsymbol{s}_2 - \boldsymbol{s}_1}{\boldsymbol{t}_2 - \boldsymbol{t}_1} , \frac{\boldsymbol{s}_3 - \boldsymbol{s}_2}{\boldsymbol{t}_3 - \boldsymbol{t}_2} , \ldots \frac{\boldsymbol{s}_{\boldsymbol{n}} - \boldsymbol{s}_{\boldsymbol{n}-1}}{\boldsymbol{t}_{\boldsymbol{n}} - \boldsymbol{t}_{\boldsymbol{n}-1}} \right)$$

where **S** and **T** are paired sequences of **n** values

The proportional derivative, or exponential rate of a sequence is a sequence of ratios of the successive differences to their starting value.

**(5.3)**

$$\frac{d\boldsymbol{S}}{\boldsymbol{S}} = \left( \frac{\boldsymbol{s}_1 - \boldsymbol{s}_0}{\boldsymbol{s}_0} , \frac{\boldsymbol{s}_2 - \boldsymbol{s}_1}{\boldsymbol{s}_1} , \frac{\boldsymbol{s}_3 - \boldsymbol{s}_2}{\boldsymbol{s}_2} , \ldots \frac{\boldsymbol{s}_{\boldsymbol{n}} - \boldsymbol{s}_{\boldsymbol{n}-1}}{\boldsymbol{s}_{\boldsymbol{n}-1}} \right)$$

where **S** is a sequence of **n** values

One variation of interest is the derivative of a sequence with respect to one of its own internal structures. If the sequence included a cyclic bias component that distracted from the subject behavior, for example, the derivative of the sequence with respect to the bias component will remove it. The result will filter out the structure of the bias. The best available mathematical description of the behavior, or different scales of behavior, could also be filtered from the data to leave a data sequence highlighting the differences between the two.

[i]eg. R. Courant and H. Robbins 1978

**(5.4)** where **S** is considered as a function of bias **B**

$$\boldsymbol{S} = \boldsymbol{S}\big(\boldsymbol{B}(\boldsymbol{T})\big) \text{ then}$$

$$\frac{d\boldsymbol{S}}{d\boldsymbol{T}} = \frac{d\boldsymbol{S}}{d\boldsymbol{B}} \cdot \frac{d\boldsymbol{B}}{d\boldsymbol{T}} \quad \text{and} \quad \frac{d\boldsymbol{S}}{d\boldsymbol{B}} = \frac{d\boldsymbol{S}}{d\boldsymbol{T}} \div \frac{d\boldsymbol{B}}{d\boldsymbol{T}}$$

or

where **S** is considered as a product of **R** and **B**

$$\boldsymbol{S} = \boldsymbol{R}(\boldsymbol{T}) \cdot \boldsymbol{B}(\boldsymbol{T}) \text{ then}$$

$$\frac{d\boldsymbol{S}}{d\boldsymbol{T}} = \boldsymbol{R}(\boldsymbol{T}) \cdot \frac{d\boldsymbol{B}}{d\boldsymbol{T}} + \boldsymbol{B}(\boldsymbol{T}) \cdot \frac{d\boldsymbol{R}}{d\boldsymbol{T}} \quad \text{and} \quad \frac{d\boldsymbol{R}}{d\boldsymbol{T}} = \frac{\frac{d\boldsymbol{S}}{d\boldsymbol{T}} - \boldsymbol{R}(\boldsymbol{T}) \cdot \frac{d\boldsymbol{B}}{d\boldsymbol{T}}}{\boldsymbol{B}(\boldsymbol{T})}$$

where **S** and **T** are paired sets of **n** values

and **B** is a paired set of **n** values to be extracted

One of the more interesting features of the algebra of natural sequences is that the full set of equivalent derivatives of any sequence is very large. Figure 2 shows one illustrative construction of a derivative hierarchy from a simple five point data set. The result is consistent with the general principle of physical continuity and provides a special case of some interest. A great many variations of this kind could be constructed to that could all be reintegrated to produce a curve passing through the same points as the original data.

In this case the underlying derivatives were constructed with successive derivative interpolation, derivative smoothing, differencing and then artificial trend separation and compression. This construction displays each underlying reversal of accelerations as possibly having developed and decayed by its own finite and separate beginning and ending processes, each as a complete and independent event in itself.

Though the example is a hypothetical construction, it illustrates that many complex hierarchies of process events could be consistent with the same simpler appearing data. Thus we would have to assume that any individual event could have any of a variety of complex underlying structures, and still have much the same outside appearance.

The example also illustrates some of the kinds of structural information that could be looked for and suggests the importance of having a quality of raw data capable of displaying a high level of detail. It also raises an interesting mathematical question. What is the simplest form of constructable real valued series that is consistent with the principle of physical continuity? It would seem possible that any form of derivative hierarchy representing an individual transition event that does *not* decompose into individual events would have regions where the scale of underlying derivatives would tend to infinity.

The example also suggests a potential for investigating the causation of events in a new way. You might find synchronized events in the second or third derivatives of, say, health care costs and rates of health club participation, or spark development and surface ionization, for examples. If that was followed by a mismatch in the timing of underlying events, you could establish a probability that the two were independent trends with the same identifiable origin. This kind of reasoning could be applied to subjects of any other kind as well, perhaps cellular metabolic cascades or the evolution of plasma states, etc.

Figure 2 also shows periods of monotonic growth (when all derivatives have the same sign) as implied by the general principle of continuity. The monotonic periods implied for the original data are visible at the very beginning and end of the first derivative curve. Their shape and duration are contingent upon the underlying derivative events. At each higher derivative level the number of events increases and the apparent durations of the first beginning and ending monotonic growth periods shrink. Additional periods of monotonic growth appear at the beginning and end of each of the underlying events of each higher derivative level.

This considerable complexity that must be presumed possible within the representation of an individual event with time-series data illustrates that the general principle of continuity is relatively weak in terms of defining the particular process structures of individual events. The strength it has is for opening a door to a relatively uncharted territory of natural structures.

## *6.* EXAMPLES

Two concrete examples were prepared for this discussion, a recording of a simple physical motion, and the historic record of US economic product. Some interesting structure will be identified in each subject, but the primary purpose here is to display the methodology and analytical tools and not any particular results of using them. A further demonstration is provided in Appendix B showing matching results with an advanced statistical method of time-series analysis.

The recording of a simple physical motion was made of a hand movement recorded using a software utility that tracked the locations of the cursor on a computer screen. The recording started and stopped when the changes in position first exceeded and then fell below a pre-set threshold. An effort was made to capture the simplest single motion possible. The historic record of US economic product was taken from the Dept. Of Commerce historical abstracts and includes the full record of statistically reliable aggregate data.

### *6.1 The Dynamics of Simple Hand Motion*

Figure 3 shows a typical group of hand movement recordings without enhancement. Figure 4 shows the derivative reconstruction and differencing of one of these and Figure 5 the same steps of reconstruction applied to a contrasting record for comparison. The curve selected for use in figure 4 was chosen based on its apparent recording of a simple single movement impulse. The intent in displaying these curves is to show what can be found in a data record with very few points. The list of analytical operators used is shown below the graphs.

In Figure 4 the reconstruction of the implied physical continuity was developed in three steps, two steps of derivative interpolation (din), inserting points so as to minimize the implied derivatives, followed by derivative smoothing (ddsm). The record data has 9 points and the dr interpolation 36 points. This reconstructed approximation of continuity was then used as the basis for calculating the derivatives.

Each of the derivative curves in this case was calculated using simple differencing of adjacent points combined with scaling for presentation (dif-2x0.xxx). The scale factor applied scales the resultant curve so that its peak value is 4/5 the peak value of source curve. Derivative smoothing (ddsm) was also applied after each successive differencing. The algorithm adjusts the midpoint of a moving 5 point bracket so that the local third derivatives before and after are made equal and the local fourth derivative zero. The fourth derivative of the full record does not become zero because different groups points with different third derivatives are used at each step. At the beginning and end of the curve assumptions about the missing points in the bracket are used.. The combined procedure provides strong and gentle smoothing as evident in the flowing character of the derivative interpolation curve (2) without departing from the original data points, but it also introduces some subtle irregularities. One of these can be seen in the slope of the derivative interpolation during the first data period. It has a slight negative slope. This results from using an assumption in the end point algorithm that projects a damped continuation of future derivative trends into the past. This is a good assumption in most cases, but not, as in this case, when there is clear reason to believe that future derivative trends did not continue prior to the beginning of the data.

These effects are undesirable, and often avoidable, but for this presentation it was decided to not adapt the algorithms for each purpose and instead to use only one standard group of reasonable settings for all subjects. A more stubborn group of irregularities introduced by the derivative smoothing procedure have to do with a tendency of the fairly simple algorithm used to introduce new second derivative inflection points when the source curve is either too rough or too regular. Successive derivatives would magnify these irregularities greatly. This is the reason for multiple applications of derivative smoothing (ddsm) at each derivative level.

The general shapes of the higher derivatives are still believed to be substantially accurate representations, given that there is no procedure that will make up for the tendency of successive differencing to quickly exhaust the information content of the raw data. The derivative level at which the information in the data has been exhausted varies from sample to sample and seems best indicated not by mounting uncontrollable irregularity, but by successive derivatives failing to suggest new pattern or to expose levels greater complexity . In statistical time-series analysis data is commonly differenced until the remainder appears uniformly random, or ‘stationary’, and then reintegrated as a straight line with a statistical component. Though this global combination of measures from different points in time is not valid in the reconstruction of an event’s physical continuity, it seems likely that the same phenomenon and useful limit of successive differencing may apply.

The initial period of monotonic derivatives implied by physical continuity is apparent in figure 4 in the period of the first two data points Though many other curves could be constructed that would reintegrate to reproduce the original data points, this set of curves approximates the simplest possible dynamic path of the behavior and the minimal introduction of information not part of the original record. The period of monotonic derivatives also appears in the results tautologically, due to the assumption of physical continuity being used and the presumption that there was no motion (a steady state) shortly before its measurable appearance above the sensitivity threshold the of the measure. Thus the reconstructed image of underlying derivatives does not, and is not intended to, prove anything about the behavior in question, but only to create a reasonably justifiable image of the underlying processes which might be useful.

One of the features of interest in Figure 4 concerns the shape of the third derivative curve(curve 5). The third derivative curve (the implied rate of change of acceleration) quickly completes one major fluctuation, and then drifts back toward zero. The dynamic portion of the curve is nearly complete by the time the peak velocity of the motion is achieved. This seems to indicate that during this individual occurrence the period of physical impulse was very short, ending more quickly than it started, and that the period of decay following the peak was relatively long and behaviorally stable. This could be interpreted as a relatively constant third derivative during the decay period and an implication that a second degree equation of velocity could be investigated for the character of friction drag in anatomical systems. This may or may not seem strongly indicated, but is one example of the type of insight being looked for.

Figure 5 shows the reconstructed derivatives of a more common single movement event. The data still appears outwardly regular, having only the one actual slope reversal , but is shown to be much more complex in its derivatives. Though drawing implications about more complex events from limited data is less reliable, the third derivative (curve 5) might still contain useful suggestion of the subtle irregularities in the physical inpetus underlying the developmental processes of this event. .

The reliability of this method of investigation can be partly judged by comparing the scale and timing of derivative peaks constructed by other methods of smoothing, and the fit to the original data achieved by reintegrating derivatives after further smoothing. Figure 6 compares the differencing and reintegration of the raw data with these operations for derivative reconstruction and symmetric running averages. The first step for each method is to construct an image of the implied continuity by interpolating new points, either by derivative interpolation (dr) or by linear interpolation and smoothing by symmetric running average (av), respectively. For clarity these starting curves are not shown.

Given the correct integration constant, reintegrating the differences of the raw data exactly reproduces the original data (1 & 2). The timing of the peaks in the raw data difference curve is in error by one half data period due to the convention of assigning the difference between two points to the time of the second point. The smoothed differences of the av and dr curves (3 & 5) more accurately identify the timing of derivative peaks because additional data points have been introduced. The scale of the smoothed av derivative and its reintegration are both in considerable error. The scales of the smoothed dr derivative and its reintegration closely match those of the original data and its differences. At higher derivatives these discrepancies are greatly magnified. Given that this demonstrates that derivative reconstruction produces an improvement over running averages for representing the continuity of a time-series, the question remains whether the improved images created are of real things. That can only be determined in each individual case according to whether the new kinds of questions that are raised prove useful.

### *6.2 Growth Rate Trend of U.S. Economic Activity*

The history of U.S. economic product as recorded in the Dept. Of Commerce National Income and Product Accounts provides an example of the potential for surprising new results using derivative reconstruction on familiar and previously well studied data. This same data set is an important focus of study in introductory economics [i] and has been modeled extensively using conventional econometric [ii] and dynamic systems[iii] models. Yet, according to one of the more pragmatic recent overviews of the econometric methods conventionally applied, little real advance has been made (Zellner 1994)[i] Throughout the literature on the subject there is a clear sense that the investigators feel there is a pattern in the data to be found and equally clear that it has remained invisible.

What continuity analysis provides, based on the assumption of physical continuity and the homeostasis, is a greatly improved direct measure of the whole system growth trend. The use of this work would not be for making quick policy or investment decisions, nor quarterly forecasts directly, but for informing theoretical analysis and pragmatic forecasting methods. Because derivative reconstruction interprets time-series as direct

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[i](Samuelson & Nordhaus 1985) Ch 36 Economic Growth Theory and Evidence pp793 Referring to graphs of the national product accounts, “Figure 6-3 is important. Linger over it.”.
[ii]see (Kmenta 1986)
[iii]see (Winter 1994)(Meadows et al 1972)(Meadows,, Meadows & Randers 1992)

images of unique non-statistical behaviors that are mathematically too complex to model with equations, in direct contrast to the conventional methods[ii], it is understandable that interpreting these results will require some adjustment.

One of the suspicions that this analysis confirms is that the US growth rate has been declining. However, the common notion that this trend began in the early 1970's.[iii] appears to be an illusion resulting largely from the untreated data's visual appearance. The actual decline in growth rates appears to have been taking place steadily over a much longer period of time.

Figure 7 shows graphs of the Dept. of Commerce figures for annual U.S. GNP and GDP[ivv]. The measures have been scaled in constant 1958 dollars. The GNP data includes foreign earnings of U.S. citizens and is no longer used as the principle measure of the US domestic economy due to the globalization of the economy. The more recent figures available are for Gross Domestic Product. In order to make a curve covering the whole period these two measures have been cut and spliced at 1960, a point in the middle of a period of when the two measures were nearly coincident. Some of the easily recognizable features of the graph are its general upward sweep, the major fluctuations of the 30's and 40's involving the great depression and World War II, and the staircase of recessions since the 1970's. For comparison a pure exponential (constant growth rate) curve is shown that seems to nearly match the historic trend until the seventies. This is the simple observation from which economists get the impression that the economy has had an underlying 'growth constant' which ceased operating for some reason in the 70's.

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[i](Zellner 1994) p218 "Most 'one shot' attempts to model this variable.(GNP).have failed...many large-scale model forecasts (are) not as good as those of very simple univariate naive models."
[ii](Kmenta 1986), pp 207 "In econometrics we deal exclusively with stochastic relations"
pp 203 "In fact the entire body of economic theory can be regarded as a collection of relations among variables..(as defined)..by a given equation"
[iii](Samuelson & Nordhaus 1985) pp 799 "the rapid growth in output per worker came to an abrupt halt in 1973" "after growing at 2 1/2% annually from 1948 to 1973, labor productivity grew at the much slower pace of 1/2% annually from 1973 to 1984"
[iv]National Product and Income Accounts
Bureau of Economic Analysis, U.S. Dept. of Commerce ;
NIPA 1869-1970 Gross National Product in 1958 Dollars Series F 1-5 , with the initial figure for 1869-79 treated as an average centered on in 1874
NIPA 1929-1982 Gross National Product in 1982 Dollars Table 1.2 (adjust 1958$)
NIPA 1929-1988 Gross Domestic Product in 1987$ (adjusted to 1958$)
[v]World Almanac 1994, Bureau of Economic Analyses, U.S. Dept. of Commerce
NIPA 1990-1993 Gross Domestic Product p104 (adjusted to 1958 $)

Figure 8 shows the construction of a long term trend using derivative smoothing and inflection point bridging. The first feature to note is that the trend curves thread through the fluctuations. The Long Periods Trend (3) was produced in two stages of smoothing and bridging, shown in more detail in the enlargement of the 1920 to 1960 period. The names of the operators used in the construction are listed below the graph. Derivative smoothing with two iterations (ddsm-2x2) was used first to make a more regular curve (2) within the range of error of the data to use as the basis for locating local inflection points. The Short Periods Trend (3) then results from trend bridging (tlin-1x25pts) drawing a curve through each of the inflection points in the dr smoothed data. This operator would have allowed a bridge of up to 25 years between inflection points, though in application the longest period between inflection points on this application was probably 3 or 4 years. Bridging algorithm produces straight line segments between inflection points including intermediate points corresponding to each data point in the bridge period. Continuous curvature is then restored by derivative smoothing to find the points the curve would have had to pass through if its underlying 4th derivative accelerations are minimized.

The Long Periods Trend resulted from a second stage of inflection point bridging to thread through each of the fluctuations in the smoothed Short Periods Trend, this time connecting alternating inflection points. In the enlargement, the inflection points of the Short Periods Trend used to make the Long Periods Trend are indicated by the circled dots. The uncircled dots are the inflection points that were skipped. The Long Periods Trend is smoothed again before differencing.

Using the same method to thread through time-series graphs of a swinging pendulum demonstrates the physical principles of simple gravitational motion. Connecting the inflection points its horizontal displacement (a decaying sin curve centered on the time axis), would give you a horizontal straight line coinciding with the time axis, indicating the invariance of gravity. Connecting the inflection points of a the pendulum’s vertical position (a decaying sin curve bounded by its stationary height) would yield a decay curve indicating the rate of energy loss of the system.

Figure 9 shows some of the difficulty confronted by the conventional approaches to the problem of analyzing the economic data in this way. The plot of annual growth rates (2) suggests a rate of growth wildly fluctuating about a 3.3% norm without any clear trend. Even when the data is strongly smoothed using a running average (3) the growth rate trend (4) fluctuates irregularly about the norm without obvious trend. The running average calculation (dasm) used three point averaging forward and back for a total of five iterations, the first two with proportional center weighted averaging and the last three with unweighted averaging. The growth rates were calculated using the proportional difference **Δy/Y** plotting the ratio of the change during each period to the height of the curve. If there really was a growth *constant* the growth rate curve would presumably have some tendency to parallel the horizontal 3.3%/yr proportional difference curve of the exponential graph shown in figure 7.

Figure 10 shows a derivative reconstruction of the historic trend in growth rates (3) calculated from the Long Periods Trend of figure 8. To help evaluate the reliability of the reconstruction a comparison of the calculation on overlapping periods is shown in figure 11. Here the analysis was done with look back comparisons from 1930, 1960, 1980 and 1993. In all cases the reconstructed trend during the ten year period preceding of the time of calculation diverges somewhat from later calculations. In one case (1960) the direction of the trend in the end period is significantly different from that calculated with the benefit of future data. These effects display the influences of local trends near the end of the analysis period and the degree of error that can be expected if this method were used for extrapolating future trends. Given this qualification, what results is a rather clear pattern continuing to the present. It is a pattern that, while not suggested by the direct and average rate calculations in figure 9, is completely consistent and clearly visible once you look for it. The predominant trend throughout the recorded history of economic growth has been growth rate decline.

No attempt is going to be made to explain why the economy behaved in the way this analysis suggests, but a little further discussion seems in order. Of particular note is that there appears to be no indication whatever of there being a ‘growth constant’, but rather a series of relatively stable trends. The current period of declining growth rates is shown beginning around 1960 , has been steady and shows no indication of turning. The current trend of decline might be attributed to a drag on productivity growth due to the modern burdens of crowding, complexity, resource scarcities and conflicting environmental impacts, but these conditions did not exist in the previous long period of growth rate decline. Some other source of drag was apparently operating at that time and the current source of drag may not be what we think. One way to identify a common source of drag for the two periods of decline would be to look for other measures with matching trend inflection points.

The two long periods of growth rate decline are interrupted by a period of increasing growth rates shown as spanning from 1920 to 1960. The period of increasing growth rates roughly corresponds to the development of modern heavy industry and the general integration of the sciences, engineering and education with production, along with the compelling and disruptive events of the great depression and World War II. It was also the period when government first took an active and comprehensive interest in economic affairs.

Figure 12 shows a contrasting image of the structure of the growth trend resulting from using a slightly different sequence of derivative smoothing and bridging steps (series B) as compared to the one presented in figure 10 (series A). Here the reconstruction of the growth rate (3), was made without smoothing the raw data prior to the first stage of bridging between inflection points. Otherwise the constructions were the same.

This new image of the long term trend is of interest partly because it displays the possible large scale effect of very subtle differences in the data and analysis procedure. It is also of interest because the new image lends itself to a more event driven view of history than an evolutionary one. The period of increase in the growth rate closely coincides with the period of the great depression and World War II, and seemingly little else.

The possibility that the growth rate rebounded during the 30's and 40's in conjunction with the century's two most disruptive events, and that our current trend of growth rate decline started immediately afterward in 1945, is intriguing. It would support the 'accumulating rigidity & creative destruction' model of economic cycles on a large scale. It might also portend a destruction of the rigidities in our present world order as the culmination of our current long trend of growth rate decline. There are certainly other possibilities. The natural world is full of self-organized systems of all kinds that exhibit long trends of growth and growth rate decline, and which do *not* become unstable. What would seem suggested, though, is a comparative study of systems that do and do not become unstable in the absence of growth, to see what makes them different.

It is possible that either image would stand out as dominant on further study, though it seems more likely that both were operating simultaneously and are each highlighted by slightly different adjustments of the lense through which the data is being viewed. The evolutionary view of the data was presented first based on the likelihood that initial derivative smoothing more precisely located the true inflection points in the raw data, making it more literally accurate. It was also presented first because event driven hypotheses tend to distract attention from the complex contexts of events that actually seem to dominate most processes of self-organization in natural systems.

More germane to this technical discussion is that these contrasting images point out the mathematical sensitivity of the tools used to what you might call *focusing*. The construction of time-series derivatives and the location of inflection points is useful because it distills and magnifies small consistent differences. Though the present method is disciplined and stable, focusing an image is still partly a matter of judgment and circumstance. The most frequently confronted type of problem came from small fluctuations in the data at the peak or trough of larger fluctuations. Because the fluctuations are what are used to locate the trend, fluctuations far from the trend can confuse the results. This was confronted in the economic data series during the 30's and 40's where the data contains large irregular disturbances far from the trend. In this case the problem was largely overcome by bridging the fluctuations in two stages.

The major difference between the results presented in figures 10 and 12 seems to largely rest as much on the reduction in the number of inflection points resulting from initial smoothing of the data as on changing their precise locations. A change in the number of inflection points in one period can unpredictably alter the sequence of inflection point selections in other periods when, as in the presented analysis alternating inflection points are connected for the second stage of fluctuation bridging.

The final picture of the economic data to be offered, figure 13, is of the simple first and second differences ($\boldsymbol{\Delta y/\Delta t}$ and $\boldsymbol{\Delta^2 y/\Delta t^2}$ ) of the long periods trend curve. Reading linear time derivatives is significantly different than reading proportional rate derivatives. Though time derivatives are more true to absolute scale the shapes of the curves also change with the scale of the subject. Curve group (A) is based on the dr interpolation of figure 10 and curve group (B), shown in the background, is based on the dr interpolation of figure 12.

One of the interesting features displayed is the peak of the second derivative in 1960, marking the center of a long period of increasing and decreasing absolute acceleration in economic production. This turning point is similar to the 2nd derivative turning point at the middle of a shorter period of increasing and decreasing absolute acceleration around 1890 (as shown in the enlargement). Though the earlier event looks different in context due to the changed scale of the economy since then it appears similar in scale to the economy of its time. Another interesting feature is a fairly subtle one, that the second derivative since 1960 has been steadily decaying toward zero. Even considering only the most reliable portion of the construction, ignoring the most recent 10 years, there is a curious strong appearance that expansion of the U.S. economy has been steadily approaching a linear rate of expansion.

# *7.* Conclusion

Time-series measures of change in individual subjects have been assumed to represent direct images of unique complex processes which can be studied on multiple scales of description.   A presumption of physical continuity has been shown to be an implication of the conservation of energy and to imply a necessary dynamic shape and symmetry of natural processes which appear to be commonly found in the observation of real events. Along with other simple assumptions about a physical system, such as homeostasis, logically simple analytical tools have been presented that refine and separate the multiple scales of behavior represented.   The recognition that physical continuity from one data point to the next implies a constrained hierarchy of accelerations that strongly limit the possible paths of development that a subject might have taken, seems to be successful in identifying unexpected and useful new questions about underlying behavioral structures.

The theoretical issues raised and preliminary development of new research methodologies are intended to provide a basis for further work.   Hopefully a strong case has been made for the usefulness of studying the structures of individual events in general, and the analysis of time-series data disciplined by the application of physical principles in particular.   The way nature orchestrates complex and well organized processes, often without any apparent template to follow, remains a significant puzzle. This approach offers new ways of looking for the answers.

# *8.* Acknowledgments

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1869-1970 Gross National Product in 1958 Dollars Series F 1-5 with the initial figure for 1869-79 treated as an average centered on in 1874;
1929-1982 Gross National Product in 1982 Dollars Table 1.2
NIPA 1929-1988 Gross Domestic Product in 1987 Dollars

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## Footnotes

p 1 i Concerning the close dynamic relationship between long-term growth and unemployment trends

p 4 i CFD, Computational Fluid Dynamics has been applied to natural convection problems with some success in various fields including building climate modeling and design.

p 5 i Applied Scientific Research V51 1993 “Advances in Turbulence”

ii (Weiner 1949)(Otnes & Enochson 1972)(Meisel, 1972)(Chatfield 1975)(Cuthbert & Wood 1980)(Harvey, 1981)(Kmenta 1986)(Smith 1987)(Evans 1988)(Casti 1989)(Mills; 1990)(Bendat, 1990)

p 6 i (Weiner 1949) p1 ‘Time series are sequences...studied with respect to the statistics of their distribution in time” p3 “The events which actually happen in a single instance are always referred to as a collection of events which might have happened” p11 ”Without at least an approximate repeatability of experiments, no comparisons of results at different times are possible, and there can be no science. *That is, the operators which come into consideration are invariant under a shift in the origin of time.*”(original emphasis)

ii (Harré 1972, p 182)

iii The practice of representing nature as mathematics appears to have originated with Aristotle, who wrote that ”it is manifest then that time is a number....and is continuous, for it is of what is continuous” and “....It is because the magnitude is continuous that the change is too” Aristotle’s Physics, Book IV 220a24 & 219a10

p 7 i The positivist route to validating generalizations can not be met for continuity. Continuity is a presence of information about connecting processes, and discontinuity an absence of information. Since all measures and observables begin and end with an absence of information neither continuity nor discontinuity of natural processes are *refutable*, thus the weaker test of whether a principle is *useful* is adopted here.

p 8 i see (Van Gicch 1978)(Miller 1978)(Meadows, Meadows & Randers 1992)

ii see (Looss & Joseph 1980)(Gleick 1987)(Nicolis & Prigogine 1989)(Lewin 1992)(Kauffman 1993)

p 9 i For example see (Miller 1967) pp 114-150

p 13 i Note that this is a mixed use of the term ‘derivative’ as defined in the calculus. The derivative structure in this case is implied by patterns that appear in successive differences in a finite real valued series. One might prefer to reserve the use of the term derivative exclusively for continuous mathematical functions rather than allow its use for physical systems and finite series as well. However, the author feels uncomfortable with the ‘differences’ of a series or only speaking of the ‘underlying acceleration patterns’ of natural systems, and so ‘derivative’ will be used somewhat broadly here to refer to implied or potentially constructable underlying continuities of acceleration. As usual the terms ‘differencing’ and ‘1st and 2nd differences’ will be used when referring to specific arithmetical operations. The term ‘integration’, however, will be used interchangeably due to the difficulty of using ‘additions’ or ‘periodic summation’ of a series, or other like terms that would be more specific to a use with finite series.

p 14 i AutoCAD and AutoLISP are registered trademarks of AutoDESK inc.

p 22 i Courant & Robbins 1978

p 26 i (Samuelson & Nordhaus 1985) Ch 36 Economic Growth Theory and Evidence pp793 Referring to graphs of the national product accounts, "Figure 6-3 is important. Linger over it.".

ii see (Kmenta 1986)

iii see (Winter 1994)(Meadows et al 1972)(Meadows,, Meadows & Randers 1992)

iv (Zellner 1994) p218 "Most 'one shot' attempts to model this variable.(GNP).have failed...many large-scale model forecasts (are) not as good as those of very simple univariate naive models."

v (Kmenta 1986), pp 207 "In econometrics we deal exclusively with stochastic relations" pp 203 "In fact the entire body of economic theory can be regarded as a collection of relations among variables..(as defined)..by a given equation"

p 27 i (Samuelson & Nordhaus 1985) pp 799 "the rapid growth in output per worker came to an abrupt halt in 1973" "after growing at 2 1/2% annually from 1948 to 1973, labor productivity grew at the much slower pace of 1/2% annually from 1973 to 1984"

ii National Income & Product Accounts Bureau of Economic Analysis, U.S. Dept. of Commerce NIPA 1869-1970 Gross National Product in 1958 Dollars Series F 1-5 initial figure for 1869-79 treated as an average centered on in 1874; NIPA 1929-1982 Gross National Product in 1982 Dollars Table 1.2 (adjusted to 1958 $); NIPA 1929-1988 Gross Domestic Product in 1987 Dollars;

iii World Almanac 1994; source for Bureau of Economic Analyses, U.S. Dept. of Commerce;NIPA 1990-1993 Gross Domestic Product p104 (adjusted to 1958 $)

# *Appendix A.*

# *REFERENCE TO ANALYTICAL SOFTWARE*

The computer tools described below are organized as a collection given the name of *Curve*, which includes all those used for the time-series data analysis presented along with some drawing and graphic utilities and templates. They were developed in the AutoLISP[i] programming language for use with AutoCAD[i] running on DOS or Windows computers. AutoCAD is a general graphical database product used extensively in architecture and engineering design, mapping and control applications. The Curve analysis collection operates only in an AutoCAD environment and is not directly transferable to other platforms. Any graphing or database software with a reasonable

[i]AutoCAD and AutoLISP are registered trade names of AutoDesk Inc.

customization capability could be used to develop similar tools following the basic principles outlined here.

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**Queries**: Questions from researchers and developers seeking help with programming and analysis problems are welcome. Copies of the software are available for research use.

**Data**: Interesting data for experimental analysis would be very welcome. Any well documented time series suspected of having underlying structure would be of interest. Ideal data sets would cover the entire history of some subject event from before its beginning to after its end. Data in comma separated ASCII text field format with title and comment heading lines, column headings and optional end field notes for individual data points would be preferred.

### *8.1.1 'CURVE' Programming Notes/Command List*

**1) <u>Command Operation</u>**

- Once the functions are loaded, the basic user operation sequence is to enter an analytic function name, select data graphs on the screen to operate on and then choose from various options. The programs then inspect the data and the attached history of prior operations and then scan and interpret the point values. Finally, when the analysis is complete the results are plotted and the command name and options used are added to the operations history. Some functions scan forward only and others both forward and back, using 2, 3, 5 or more points in the analytical point bracket, and may perform single or multiple iterations.

**2) <u>Presentation of Results</u>**

- Every analytic function creates a new graph, as every operation on a mathematical equation creates a new formula. When several steps are taken to produce a desired result the intermediates are typically hidden or discarded unless they present useful visual information.
- Because the scale of derivatives and integrals is often quite different from that of the original data, derivatives are sometimes automatically rescaled with a peak value of 4/5 of the peak value of the original and integrals at 5/4 of the original. When a comparison of different data sets or methods of analysis is desired, a fixed scaling factor is selected so that the results have matched presentation scales.

**3) <u>Major Functions</u>**

- Derivative Interpolation - DIN locates a new point in the middle of a four point bracket that makes the third derivatives on either side equal, using sub-function (F_3SYM). This creates a curve, including the original data points, which has greater continuity than the original and approximates a curve with the minimum scale and reversals of underlying acceleration necessary for a continuous path between the given points.

- Trend Line Bridging - TLIN- draws a graph between local inflection points as defined by reversals in the sign of the second derivative. This corresponds to the principle that fluctuations about a trend will cross the trend line with a maximum slope and have a slope equal to that of the trend when at a maximum distance from it. This is commonly the case for homeostatic processes. .

  Options are provided to select every, alternating inflection points or to prompt inflection points for individual user selection, and to set a maximum length of fluctuation to recognize.

  > The best large scale trend results took several steps. First interpolation and derivative smoothing were done and then the short period fluctuations filtered out. After additional derivative smoothing the long period fluctuations could be identified and bridged.
  >
  > If the last inflection point was close to the end of the data the end point was placed according to a weighted projection from the preceding trend third derivative using (FF_TENDS). This substantially reduced endpoint distortions.
  >
  > Each line segment of a trend bridge graph has a separate point on the bridge line for each original data point during the period. When bridge lengths were large the number of points was reduced before derivative smoothing using (GPAR). This is required to maintain a regular frequency of points and then to eliminate long periods of false constant slope which derivative smoothing would only reinforce. (See Figure A1)

- **Double Derivative Smoothing** **-** The objective of derivative smoothing is to reduce the number and scale of underlying derivatives while leaving the integral of the curve unchanged. **DDSM** combines simultaneous forward and backward scans adjusting the middle point of a 5 point bracket using third derivative smoothing (F_3SYM), or a 3 point bracket using 1st derivative smoothing, to equalize the rates of change on either side of the center point. Sometimes referred to as curve fitting, rather than smoothing, the routine does slightly effect local maxima and minima but has rapidly declining effect on repeated iteration. It provides strong local smoothing but has little effect on overall curve shape or timing.(see Figure A.2)

- **Graphic Function Calculator** **-**The **GCAL** function will perform calculations using the names of graphs and user selected points and distances as variables in equations written as text with a general set of mathematical functions. One curve is used for the time value set and the implied scale of the other curves at those points is used in the calculation. The function used can be read from and to selected lines of text and is recorded with the resultant curve as part of its operation history record . Graph variable names are identified by starting with a double letter (i.e. GG1, etc.) and points variable names by starting with 'pt' (i.e. PT1, etc.). Other preset AutoLISP variables may also be used by name. This utility can be used to produce function graphs such as exponentials or log plots of data. Its primary intended use is for applying theoretical relations between dynamic measures, such as to examine energy flow and temperature relations, etc.

**4) Subfunctions**

- **3rd Derivative Smoothing (F_3SYM)** - adjusts the scale of the midpoint of a five point sequence.. The calculation shown here assumes equal time periods. In the program the result is then adjusted for unequal periods. The equation yields the Δy (**ΔYi**) for the middle point that will make the 3rd derivative in the first three periods equal to that in the last three periods :

$$\Delta \boldsymbol{y}_{\boldsymbol{i}} = \frac{1}{2}\left( \Delta \boldsymbol{y}_1 + \Delta \boldsymbol{y}_2 - \frac{1}{6}\left( \frac{\Delta \boldsymbol{y}_3}{\Delta \boldsymbol{x}_3} - \frac{\Delta \boldsymbol{y}_0}{\Delta \boldsymbol{x}_0} \right)\left( \Delta \boldsymbol{x}_1 - \Delta \boldsymbol{x}_2 \right) \right) \qquad \text{A.1}$$

Algorithm for 3rd Derivative Smoothing
(refer to point and ratio naming convention in figure A3)

Point variables in the sequence are labeled from 0 to 4 and differences from 0 to 3. The routine also makes corrections at points where new second derivative reversals would be introduced and at the mid-points of double reversal periods which produce inconsistent sign and magnitude errors.

- (**F_DOGRAF**) The Simpler graphing functions were written to record results as each successive point is read and interpreted according to a rule fed to this graph drawing sub-function.

- (**F_DXYVAR**) and (**F_LISTVARS**) The more complex functions maintain numbered variable names for each place in the bracket being considered with data values successively moved from one place to the next as the curve is scanned.

- (**FF_TENDS**) Start and end points of a series are sometimes retained unchanged from the original curve and sometimes projected according to a damped first, second, or third derivative implied by preceding or following points..

**5) Basic Operations**

- **Importing Data Files - GRFIL** imports comma separated two column text data along with title, headings, scaling factors and notes.

- **Derivative - DIF** plots the ratio of $\Delta y/\Delta t$ with the starting point assigned a value equal to the average of the second and third. The proportional difference, or growth rate, option provided is $\Delta y/y$. The auto-scaling option sets the peak value of the derivative to 4/5 that of source.
- **Integral - INT** plots $\Sigma\Delta y*\Delta t$ the area under the curve of each period with the starting point set at a user picked or entered value. The auto-scaling option sets the peak at 5/4 of that of source.
- **Graph Scaling - GSC** scales a graph, in proportion to the ratio of one distance to another
- **Increasing Point Frequency - GSEG** (linear interpolation) inserts a variable number of points equally spaced on a straight line between existing points. Used to increase the point density of a data set..
- **Reducing Point Frequency - GPAR** Creates a subset partition of the data set by skipping points in a given range or replacing a range of points with an average. Used to reduce the point density of a data set. This was primarily applied to a TLIN curve to create an equally spaced point set that could be effectively smoothed by DDSM.
- **Average - ASM** averages a variable bracket of points, with each point given equal weight or a weight in proportion to its position in the point bracket.
- **Symmetric Double Average - DASM** is the same as **ASM,** but scans both forward and back and averages the results to reduce directional bias.
- **Derivative Smoothing - DSM** with 1st or 3rd derivative symmetry with a variable number of iterations. Same as DDSM but without combining simultaneous forward and backward scans.

**6) Other Functions**

- **Trend Separation - TSEP** is a special application tool that separates trend periods in a data sequence and inserts constant periods at maxima, minima and axis crossing points. This is done either by simply adding new points between the developmental trend periods (increasing the range), or with linear compression of the trend period so the range of the data is the same as the original.
- **Step Curves - GSTEP** creates a step curve passing through the points of any graph. It is primarily a graphic tool.
- **Graph Recording - GREC** records the accelerations of the screen pointer for one movement of the digitizer and scales the point set to fit a pre-defined window.

# Appendix B.
# **RELATED WORK IN ECONOMICS**

### *8.1.2 Econometric Time-series Modeling*

The conventional mathematical model for a time-series in economics is a simple polynomial equation adjusted to fit the data using least squares regression, combined with a statistical component. The relative difficulty of obtaining success in making predictions or defining a theoretical basis for the equations is responded to in three basic ways. There are simple mathematical models accepted because they are easily made and understood, complex and mathematically sophisticated models that offer some improvement in results but add a burden of complexity, and mixed methods that pick up whatever seems useful. Interesting current discussion of these general approaches is found in the papers by Tiao & Tsay (1994), Young (1994), Zellner (1994)and Peña (1995) in the Journal of Forecasting and elsewhere.

The most common of the practical methods is the ARIMA model, standing for Auto-Regressive Integrated Moving Average (see Chatfield (1975) and Harvey (1981)(1993). Following this method known trends are first removed from the data, then the successive differences between points are taken (giving derivative curves) until the result seems completely random. Then a straight line is fitted to the random data, reintegrated and combined with a statistical variance to make a stochastic function to represent the behavior of the system. Peña (1995) notes that one reason that twice differenced and reintegrated models have better short term predictive results is that the procedure gives the greatest weight to the value of the most recent data point, where fewer integrations have the effect of giving greater weight to prior data points. Zellner (1994) takes a somewhat arbitrary but practical approach by taking a less than perfect ARIMA and mixing in factors from the index of leading indicators. This remains simple enough to understand and is significantly better in making predictions. Tiao and Tsay (1994) demonstrate that the performance of forecasting models depends partly on whether the number of time periods ahead for which they were optimized is the same as time period ahead they are used to forecast.

The sophisticated methods start from this approach and add variations such as piecewise optimization, statistical and frequency filtering and Fourier and Hamiltonian function analysis. Tiao and Tsay (1994) propose a method of constructing a piecewise linear autoregression that is piecewise in the space of a threshold variable so that the regression groups the expansion and contraction periods for separate optimization and then recombination. Also demonstrated is the use of Bayesian inference with Gibbs sampling for to filter out of the data statistically atypical values. Young (1994) tackles the problem of statistical non-uniformity in the data using time variable parameters found by recursive estimation based on the frequency distribution of the statistical residuals. The method is quite difficult to understand but produces continuous descriptive resultant curves displaying a remarkable level of dynamic detail.

### *8.1.3 Economic Factor Theory Models*

The models based on economic factor theory, the theoretical structural relationships between technology, education, money, population, resources and public policy, etc., are also very actively pursued. One of area of strong new interest is endogenous growth theory, in which models are constructed using feedback, a structurally different kind of mathematics than the fitting of polynomials as in conventional practice. This is the same structural difference between the formulas of classical physics and the models of physical systems being developed in the study of physical system dynamics known as chaos or bifurcation theory.

In endogenous growth theory one of the interesting ideas is that growth rates might increase with increasing population, rather than decline. Having more people available to invent things that everyone can take advantage of might explain why the large societies in history have had higher rates of growth (Kremer 1993)(Schulstad 1993). In economic theory this new approach marks the significant step toward abandoning the concept of perfect competition as discussed by Romer (1994), Grossman & Helpman (1994), Solo (1994,) and Pack (1994). Following this path the interest is to provide compelling reasoning to guide theory, policy and business practice. The construction of behavioral models from factor theory equations that fit or predict the data well is currently out of reach.

*8.1.4 The large institutional models such as the Kent model and others[i] used by governments and universities may incorporate thousands of separate time-series related according to a combination of empirical and structural factor theory equations. Most are optimized to provide one, two and three quarter business projections. There may be many kinds of innovative experiments in modeling taking place in these circles. There are also large scale resource models, as opposed to business models, used by the UN and others. One of some note that is compelling but has been treated as requiring too many assumptions is the limits to growth study originally commissioned by the Club of Rome (Meadows 1972, 1992). This work is quite out of character with the majority of economic modeling efforts in that it is a serious effort to project the behavior of the world economy as a life support mechanism for a century into the future.*

### *8.1.5 A Comparison of Results: Reconstruction of Physical Continuity & Time Variable Parameter modeling*

The reason for this section is a remarkable similarity between the results obtained for the same data with derivative reconstruction (DR) and the time variable parameter (TVP) method of Young (1994). The significant finding is that the two distinctly different methods both demonstrate rather clearly that there is another level of structural information in time-series data that has not been made visible by other methods. It also serves to confirm the validity of each method of exposing it.

Figure B1 shows the DR interpolation and 1st differencing of $\log_e$(GNP) and $\log_e$(UN) for the same data used by Young. The derivative trends of the stock market were also calculated and are presented for comparison.

[i] See Journal of Business Forecasting Methods & Systems and Journal of Monetary Forecasts

Figure B2 shows the 2nd derivative of these variables presented in the same manner as Young's results, shown below in Figure JOF-11. for the 2nd differences the filtered trend GNP and Unemp.

The remarkable feature of both is an apparent tight symmetric synchrony between the underlying accelerations in GNP and Unemployment rates, despite the appearance that the original measures describe entirely different kinds of behavior.

1) **The data used**

- Young used 160 quarters of US GNP, Unemployment rates and other aggregates covering 1948(1) to 1988(4) (figure JOF-1. And JOF-4)[i].
- For derivative reconstruction these charts were digitally scanned and converted from TIF to DXF vector graphic files. The vectorized scans were then corrected by hand to join broken segments and trim out spurious vertical segments to make them suitable for derivative reconstruction using the functions of CURVE .

2) **Analysis Method**

- Young's method uses a recursive spectral density filter called IRWSMOOTH to produce a time-series trend. It is based on the unobserved component models (referred to Harrison and Stevens 1971 & 1976; Kitagawa 1981; and Harvey 1984) in which the parameter variations are described by a higher-dimensional, vector random-walk-type model. Though the details of its construction are not presented, and its application quite complex, one mentioned feature that might cause it to have results similar to DR is the reported similarity of the state-space algorithms used to the optimization technique known as *regularization* which includes constraints on the rates of change of the variables(see Young 1994 p. 181). The IRWSMOOTH trend series was then differenced twice to produce figure JOF-11.
- In this application the derivative reconstruction steps were oriented to examining more detailed fluctuations than those used for the analysis of the entire history of GNP presented in the preceding paper. The steps began with derivative interpolation (din) to reinforce the short term events. Then nearly identical stages of derivative smoothing, inflection point bridging and further derivative smoothing were applied to each curve. The two levels of trend bridging for the unemployment figures are clearly visible in figure B1. The larger scale trends were then differenced to produce the neat mirror symmetry of fluctuating first derivative rates of the DR trend levels of GNP and Unemployment. A second differencing and scale adjustment produced Figure B2.

3) **Discussion**

[i]Figures reproduced from Journal of Forecasting for research only

- The question, of course, is whether this kind of remarkable pattern is real. It seems to have been hard for Young's readers to accept it. Young provides a careful response to the question of the relationship being just an artifact of his unusual spectral frequency filter. He also points out that there is "more than just a similar frequency content in the series: even subtle temporal variation in the cycles can be discerned in both series. Moreover, similar filtering operations applied to some of the other series (in figure JOF-4) do not reveal nearly so clear relationships" (Young 1994 p 203).
- One additional aspect of the pattern of close synchrony between underlying turning points that seems particularly convincing concerns an implication about the general system structure of the economy. Neither GNP nor unemployment are leading factors, but turn simultaneously. This seems to be possible only for measures of a system that acts as a uniform whole. If either one were the consequence of the other then a consistent time lag should be evident. Thus the synchrony of the turning points suggests that the two factors are not causally related, but are both indicators of the same dynamic of the whole system.
- The DR stock market trend derivative shown in figure B1 bears some similarity to the GNP movements, but sometimes leads and sometimes follows, in an irregular fashion. It therefore appears to be only loosely tied to the underlying fundamental dynamic of the system as a whole. This is just what one might expect considering the strong influence of volatile and self fulfilling investor expectations in setting the directions of the market.
- All in all, what seems demonstrated are two lenses with slightly different focus and lense distortions, providing clear views of the same surprisingly systematic behavior.