A Continuity & Divergence Principle
(the physics of 'little bangs' and 'big booms')

Note: This is represents a theorem developed in 1993 and finalized in a 1995 paper[1]

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1. Continuity and Divergence

The form of mathematics needed for the physical sciences to become capable of exploring emergent phenomena in nature is approached here by asking what kind of expression would be needed to describe events that begin and end.   For continuity, the conservation laws then require change to proceed by 'little bangs' followed by 'big booms'.

The constraint of energy conservation can be shown to require derivative continuity in rates of change and for beginning or ending energy flows, and for that to then require divergent sequences to do it, eruptions of change one could call "big booms".   It appears to be for much the same reason that a period of inflationary change needed to be hypothesized to start the "big bang" of the universe. When you look for them you often easily find them. Emergence and development may be the larger part of change, disguised by our bad habits of thought in ignoring what simple "cause and effect" doesn't explain.

Many things in life and emergent phenomena such as organisms, storms or sparks etc, involve energy flows and other kinds of conserved change with definite beginnings and ends. Direct observation of those transitional periods of change, to the limits of our observation, generally reveal explorable processes of conserved change that diverge from anything else around them, not following cause & effect but emerging and developing locally.   They are almost never described that way by conventional science, though, for lack of a form of arithmetic to do it.   We don't know how to connect divergent sequences with convergent ones.   We don't know how to represent behaviors with no reliably conserved properties. It's a hurdle that has often been considered impossible or unscientific to even address.   This

approach opens a door for studying them through their conserved continuities of change instead.

What seems to connect divergent processes in nature, missing from mathematics, is the environment in which they develop. Processes in an environment develop independently until they run into something else, i.e. have a form of connection without prior determinism. In that context the apparent finality of such beginnings and endings combined with the continuity required for energy conservation can be shown to imply an unlimited hierarchy of physical system scales is needed. It raises new questions, beyond our evidence, in a way consistent with the evidence at hand and pointing to where new discoveries can usually be made to confirm it. It remains an open and interesting debating point whether the apparent finality in beginnings and endings in nature, to the limits of our evidence, is genuine. Perhaps only a preponderance of the evidence and the relative simplicity of the alternate explanations can provide a satisfying answer.

The general mathematical questions of describing nature with divergent mathematical series may have been most thoroughly studied by Robert Rosen. His short 1995 essay[2] seems to clearly identify the use of mathematics absent 'improper' divergent sequences as central to his complaints about the scientific method. He notes that by choosing to study only the behavior of convergent mathematical series science arbitrarily discards the study of divergent ones. In his observation, both emergence in complex systems and in life are subjects only seen in divergent processes which science would need to use divergent sequences to study.

The present theorem demonstrates that the beginnings and endings of energy flows requires divergent sequences to be described mathematically. That identifies a key feature of life and emergence that a study of divergent sequences is needed for, substantiating Rosen's complaint. Because defining environments for equations may prove difficult and because the divergent processes of interest already have their own environments… a switch of method seems called for. Instead of using math in isolation, a way to use it in reference to open system environments seems required. One general way to do that, for example, might be to replace environmental parameters with strategic queries. The greater immediate usefulness of the theorem is offering mathematical signatures that can be used to identify divergence in physical processes that are inevitably going to run into their own environments and be changed by it. It raises new answerable questions about emergent processes of great interest.

2. Limits of Change

The principle of energy conservation, that energy can not be created or destroyed but only moved from place to place will be shown to imply that such transfers can not occur instantaneously. That implies a requirement for derivative continuity in both physical motion and other energy transfer processes. It also forms a general implied requirement for continuity in organizational change for energetic physical systems, because energy transfer processes use the organization of physical systems to operate. Organizational change in open systems seems generally indefinable and unmeasurable because it's distributed and often embodied in passive environmental potentials that are hard to identify or measure. That's what is usefully exposed by identifying the form of continuities connecting the dots.

The demonstration that divergent sequences are required to enable physical processes to begin or end with continuity begins by presenting the basic conservation laws as a hierarchy. The conservation of energy, the conservation of momentum and the conservation of reaction forces are related as derivatives and integrals of each other, one law stated differently for scales, velocities and accelerations of change. That one law can be represented as an infinite hierarchy of successive derivative laws. The familiar statement of the three basic physical laws is shown in the first three equations in column <u>a</u> of table 2.2 "Conventional Form". They are repeated in column <u>b</u> "Unified Form" altered by substituting derivatives of distance (**s**) for acceleration (**a**) and velocity (**v**), and in the case of energy conservation, the conventional term $\frac{1}{2}v^2$ is replaced by the integral of its derivative ($\int v \cdot dv$), a quantity having the same derivative rank as distance (**s**). They all have the same form of statement; that the sum of each of the derivatives of energy does not change. The general principle of continuity is then derived by successively differentiating as a limit and concluding that the sums of all derivative rates of energy flows within a closed system are conserved.

To this point, little has been said about what is in the 'closed system' and how it might relate to the open systems in which we observe the behaviors of life and other things to begin and end. If within the closed system there are visible and invisible regions, with energy appearing in one place from an unobservable source, the conservation laws tell you little about the bounding quantities of energy available. They do tell you something about the bounding rates of change in energy flow though, which turns out to be quite useful. The issue leads toward discovering how to identify behaviors exhibiting temporary conservation of organizational change, and how to use it as a temporary stand-in for energy flow. In practice one very frequently has sound evidence that change is being conserved in a system

but no good information as to where or how. Determining whether the system is displaying divergent or convergent developmental change offers a starting point for exploring that.

Column $\underline{c}$ in table 2.2 "Limiting Rates" lists physical limits of energy transfer, starting with the speed of light as the limiting velocity in line $\underline{2}$, $v_j < c$ (3.2.2c). Because it takes time for a derivative to accumulate change in an integral, as for an acceleration to change a velocity, the limits of one rate applies to the others. That is shown in table 2.1 , as follows:

For: **i, j, l, n** - integers; $\mathbf{k_i}, \mathbf{c_i}, \mathbf{u_i}$ - constants; **c** speed of light

m – mass; **a**-acceleration; **v** - velocity; **s** - distance; **t** - time

r-rate; **Δ**-finite difference; **d** – differential

2.1 Relation of Limiting Rates

If, at the n'th derivative level	$r_n < c_n$	2.1-0
in any finite period	$r_n = r_{n+1} \cdot \Delta t + k_n$	
by substitution	$r_{n+1} \cdot \Delta t + k_n < c_n$	
and	$r_{n+1} < (c_n - k_n) \div \Delta t$	
let	$(c_n - k_n) \div \Delta t = c_{n+1}$	
so that		
at the n+1 derivative level	$r_{n+1} < c_{n+1}$	2.1-1

2.2 Laws of Conservation and Continuity

	a) Conventional Form	b) Unified Form	c) Limiting Rates
1. Conservation of Energy • sum of energies is constant • 0 derivative level	$\sum_i \frac{1}{2} m_j \cdot v^2{}_j = k$	$\sum_i m_j \int v_j \cdot dv = k$	$s_j < c \ \cdot t + k_1$
2. Conservation of Momentum • sum of momentums is zero • 1st derivative level[3]	$\sum_i m_j \cdot v_j = 0$	$\sum_i m_j \frac{ds_j}{dt} = 0$	$v_j < c$
3. Conservation of Reactions • sum of forces is zero • 2nd derivative level	$\sum_i m_j \cdot a_j = 0$	$\sum_i m_j \frac{d^2 s_j}{dt^2} = 0$	$a_j < c_2$
4. Unnamed • Sum of 2nd accelerations zero • 3rd derivative level		$\sum_i m_j \frac{d^3 s_j}{dt^3} = 0$	$r_j < c_2$

5. Principle of Continuity
- Sum of higher accelerations zero
- n'th derivative level

$$\sum_i m_j \frac{d^{n2} s_j}{dt^n} = 0 \qquad r_{j_n} < c_n$$

3. Divergence in Individual Events

We now consider some individual energy flow within an open system S.   You might represent that as the movement of a mass (m) which begins at rest.   A finite force (f) to move it can't be applied instantaneously because that would imply a step change in acceleration, and an infinite force (2.2-2c), as well as the higher rates of change than allowed by the general principle of continuity (2.2-3c,4c,5c).   For it to develop any positive velocity its acceleration will need to have been positive for a finite period.   The same is then true for it to achieve a positive acceleration, it's rate of increasing acceleration, and every other underlying acceleration, will need to have been positive for finite periods.   For there to be a change from rest to motion every underlying acceleration needs to have been maintained for a prior finite period during which all underlying rates are positive.   If they all start at zero and none can be infinite acceleration is not possible.   The implication is that accelerating anything from rest is either: a) impossible, b)requires energy conservation to not always apply, c) that nothing begins or ends or is ever at rest, or d) doing so requires a trick.   One of the plausible 'trick' ways to resolve the contradiction is for things that do begin and end to do so with divergent accelerations, a burst of development or "big boom".   It would then be a demonstration to observe divergent accelerations and bursts of developmental change where motion or other energy transfer systems appear to begin and end.

One class of mathematical functions that has derivatives of the same sign for finite periods and also closely associated with physical processes resulting in bursts of organizational change and energy release, are the exponentials.   They don't quite satisfy the requirements, though, for not having any point of beginning or ending.   They can only be arbitrarily started and stopped with discontinuities that would violate the conservation laws.   What's needed then for both change and continuity is an emergent exponential-like progression of some kind, appearing at each observable scale to begin with an implicit but possibly unobservable seed of change on a smaller time and energy scale.   That sounds a little fantastic, perhaps. Because the proof is an exercise in narrowing down the difference between what needs to be found and what is generally found, all that needs to be demonstrated here is scientifically useful progress in doing that.

For example, a fire may start with a spark, definitively, but that start may be unobservably small and brief relative to the scale and course of the fire. Every scale of organization requires a different mode of description, because they each make different sense, and so it is rather natural for each mode of description to leave out the others. Why each different mode of description leaves out the others is open to question, of course, but it could be a property of how we describe things, of our own mental models, rather than of the things being described. Continuity of change appears to imply that every scale of behavior requires other scales for their beginnings and endings to occur. This principle that continuity seems to require invisible scales of behavior is not well recognized even if we do commonly see smaller undescribed functional scales of behavior in most kinds of behaviors. We also commonly see exponential-like progressions at the beginning and end of all kinds of systems and processes seeming to have definite beginnings or ends. It's possible that it just means that each scale of organization needs its own separate process of development, another implication one could look for confirmation of.

The polynomial form of an exponential function directly results from the successive integration of a constant. A starting point is provided by an assumed event of a different kind on a smaller time and energy scale, providing a "seed" for a divergent process and the "little bang" of explosive development to begin a larger system from "next to nothing" to satisfy the conservation laws. Oddly, this "unhidden pattern" is clearly visible in large classes of systems and their way of linking scales of organization, like fertilization for reproduction, or a spark to start a flame or an idea to start an industry, displaying divergent processes in-between. This way of connecting scales of organization makes it theoretically possible to have smooth change with definite beginnings and ends. The proof is as follows.

> For some large **n,** the **n**th derivative rate r_n is taken as finite and between some lower and upper bound pair of constants representing the limiting propagation rates for the process of energy transfer:
>
> $$u_n > r_n > l_n \qquad\qquad 3.1$$
>
> Integrating the **n**th derivative rate with integration constant $\mathbf{c}_{n-1}$ also chosen between some upper and lower bound limits of propagation rates for the process at that level of acceleration:
>
> $$r_{n-1} = \int r_n = r_n \cdot t + c_{n-1} \qquad\qquad 3.2$$
>
> In general, as the number of derivative levels **n** increases and the number of times $\mathbf{r}_n$ is integrated i equals **n** the form of polynomial expansion approaches that of an

exponential.

$$f(t) = r_0 = \frac{r_n}{(n-1)!} \cdot t^{n-1} + \frac{c_{n-1}}{(n-2)!} \cdot t^{n-2} + ...c_{n-i} \qquad 3.3$$

One of the further directions of exploration is to establish that there are particular upper and lower bound propagation rate limits, $\mathbf{u}_n$ and $\mathbf{l}_n$. The universal limit used in 2.1 above to establish the form of sequence required is the universal propagation rate limit of the speed of light. For any particular energy transfer process the starting 'seed' acceleration would not be arbitrary, but would have limits defined by the process itself, somewhere between the highest and lowest potential propagation rates for the larger scale process being considered. For example, the bounding limits for propagating a fire are what break the chain. At too high a rate of propagation a flame becomes an explosion and blows itself out. At too low a rate a spark cools before igniting anything else. Just looking for how that principle applies to any given process of beginning tends to be quite informative. It provides a way to follow a lead and explore the whole domain of behaviors in which the process develops.

With most observed event processes their beginning displays an exponential-like period rather than a simple exponential. There's no constraint in the above analysis requiring complex systems developing at constant rates, just that they be bounded within natural limits. Perhaps the more surprising result is the reverse implication, those organizational processes in nature identified by the divergent way they conserve their own accumulations, identify the emergence of conserved organization as a means of transforming energy, and a transitory form of energy themselves. Where such questions lead may not be immediately clear, but a path for exploring them is provided.

[1] Philip F Henshaw 1995 (Excerpt) "Reconstructing the physical continuity of events"; link http://www.synapse9.com/drafts/ ReconstCont-95_08.15.pdf

[2] Robert Rosen 1996 "On the Limitations of Scientific Knowledge" in On the Limits to Scientific Knowledge, John Casti & Anders Karlqvist eds, Perseus; collecting ten papers presented a 1995 Stockholm workshop of the same name sponsored by the Swedish Academy of Sciences; link to scanned copy http://www.synapse9.com/ref/Rosen_On_Limitations_of_Sci.pdf

[3] Basic Related Formulas of Work for reference

$$W = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m \cdot \left(\frac{ds}{dt}\right)^2$$
Work, Energy of accelerating a mass to a velocity

$$F = ma = m \cdot \frac{dv}{dt} = m\frac{d^2 s}{dt^2}$$
Force corresponding to acceleration for a mass (a first derivative of Work)