Features of Derivative Continuity in Shape

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Abstract

Derivative continuity is a distributed invariant relationship between parts of flowing shapes. The original techniques presented here were developed for making the behavioral dynamics of complex processes more recognizable, but are equally applicable to assisting in the recognition of shapes in images. Regularizing a sequence using a constraint of derivative continuity is equivalent to using a bimodal smoothing kernel, producing a distinct bias for reducing variation on higher derivative levels, sharply defining shape with minimal suppression of shape. To help determine where reconstructing shapes this way is valid a test was developed to help distinguish combinations of noise and smooth shapes from random walks. This helps distinguish between illusory and genuine data shapes, but also exposes a flaw in using this and other measures of scaling behavior for diagnostic purposes. Gaussian scale space techniques in use for some time for identifying reliable landmarks in the shapes of outlines, are demonstrated for use in identifying key features of shape in time series.

Key Words: PATTERN RECOGNITION, DERIVATIVE CONTINUITY, REGULARIZATION, RECONSTRUCTION, SMOOTHING KERNELS, CONNECTILES, FRACTAL EXPONENT, SCALE SPACE .

1 Introduction

In a literal sense, equations that represent physical behaviors are simplifications of the shapes of the data gathered about those behaviors, converted to a more useful form. That form may be more useful because of its compactness and ease of manipulation, but also frequently for seeming to embody the very structure of the behavior whose shape it imitates. The methods reported here concern one of the invariant relationships between near bye points in equations with smooth shape at the core of the special relationship between equations and nature, derivative continuity. The property is used here, , to regulate non-parametric representations of shapes with the object of better representing underlying behavior and its differential structures. That may be preferred over the use of equations for the purpose in preliminary investigations, when the subjects of study are uniquely individual or transient, or for identifying natural markers of shape or behavioral structure for object or process recognition.

Part of the underlying effort was to define continuous derivatives for sequences, using the mathematical definition of the derivative and parsimony to construct smooth curves passing through given points by iteration. For mathematical functions, having a slope at a point, a derivative, is determined by whether the slopes, between a given point and points on the curve approaching it from opposite directions, come to the same value. This definition and test for a derivative has been well worked out for mathematical functions for a long time, originally developed by Newton and Leibnetz in the invention of calculus[4]. Stated simply, curves which have a slope at every point change by continuous progressions in direction rather than discontinuous steps in magnitude, and represent distributed structures of connection.

Derivative continuity, interestingly, is not defined for the smooth flow of physical processes, except in equations representing them, nor for smooth shapes in data. For physical shapes and processes the connectedness of smooth flows is a presumption, one that has proved to be singularly useful in allowing the representation of physical behavior with equations. There are, of course, disjointed, random, fractile and chaotic patterns that seem common as well. In fact all things and processes are so full of gaps as to seem perfectly discontinuous in ultimate construction. Still the great success of presuming continuity, provides direct support for considering it as an invariant physical principle of meaningful shape, even if not well understood. Except for behaviors related to the physics of the conservation laws, the physical basis for derivative continuity in nature is relatively unstudied.. It is this study of the physical basis of continuity that has motivated much of this work. Progress on that wider interest is fairly limited but it has produced some techniques that should be independently useful.

1.1 Related Issues and Methods

The object of generalizing data shapes is to create features that distinguish the various subjects represented in scattered points of data. This requires applying constraints to what would otherwise be 'ill-posed' problems, applying various assumptions regarding the non-accidental character of nature as discussed by Wechsler[17] in relation to the problems of computer vision and artificial intelligence. Derivative continuity is typically part of the constraint applied in the use of Lagrange multipliers[17], for example, as well as in fitting 'least energy' surfaces such as with the level set approach[18], because the constraints applied are continuous functions. Continuity can also be applied using constraints on local connections between points that are weaker than optimized equations, with the benefit of preserving subtle and transient characteristics of shape that would otherwise be lost.

To determine whether it is appropriate to assume underlying continuity and the validity of apparent statistical trends or shapes, variation scaling and range tests[3,5,11] have been used. These same tests underlie the measures of fractile dimension for natural scaling patterns and have been used to identify a wide range of apparent fractile patterns in nature. One of the invariants of natural data, however, is a presence of mixed signals, originating from mixed sources. That all variation scaling and range tests are compromised by sensitivity to mixed signals has been noted[16] but is not widely investigated. Because some the measures display differing sensitivity to different data structures, however, better ways to separately identify components of mixed types of data may be developed.

Once meaningful data shapes are found it is common to use non-linear operators to isolate identifiable landmarks[8]. The Gaussian scale space methods[7,12,13,15], in simple terms, use inflection points on a curve that are relatively invariant to smoothing as identifying marks of shape. In concept, though not yet in practice, the location and kind of derivative inflection points on a smooth shape can be read like a bar-code, uniquely identifying given subjects. Given sufficiently accurate representation of shapes, various other details of the general differential structure can also be tested for robustness and used as identifying marks. While most work done in the area concerns the outline shapes of objects in visual images the method is also directly adaptable to identifying scales and landmarks of differential structure in time series.